18.06 (Fall '12) Problem Set 10

This problem set is due Thursday, December 6th, 2012 by 4pm in 2-255. The problems are out of the 4th edition of the textbook. For computational problems, please include a printout of the code with the problem set (for MATLAB in particular, `diary(“filename”)` will start a transcript session, `diary off` will end one, also copy and paste usually work as well.)

1. Do Problem 6 from 7.1.

   (a) Does not satisfy either property. Take $v = (1, 0)$ and $w = (1, 0)$. Then $T(v+w) = T(2v) = (1, 0)$, but $T(v) + T(w) = v + w = (2, 0)$.

   (b) This satisfies both properties, as it is linear in each coordinate.

   (c) Again, since each coordinate of $T(v)$ is linear in each of the inputs, it satisfies both properties.

   (d) Does not satisfy $T(v + w) = T(v) + T(w)$. Take $v = (1, 0), w = (0, 1)$. Then $T(v + w) = 1$ but $T(v) + T(w) = 1 + 1 = 2$.

   If "largest" means "largest in magnitude" then it clearly satisfies $T(cv) = cT(v)$. If "largest" means "maximum" then it does not; consider $v = (1, 0), c = -1$. Then $T(cv) = 0$ but $T(v) = 1$.

2. The $n*n$ matrices form a vector space of dimension n^2. Let A and B be two matrices in this space. Which of the following map from the space of $n * n$ matrices to itself or to $\mathbb{R}$ are linear ? Explain your answer.

   (a) XB is linear since by definition of matrix multiplication each entry of XB is just a linear combination of the entries of X. Similarly, AX is linear. Since a composition of linear transformations is linear, we also have AXB is linear.

   (b) This is not linear. Consider any non-zero X. Then $(2X)^T A(2X) = 4X^T AX \neq 2X^T AX$.

   (c) Yes, AX and XB are linear as before, and the sum of linear transformations is linear.

   (d) Yes, the trace is just a linear combination of the entries of X.

   (e) No. Consider the two-by-two identity matrix I. Then $\det(2I) = 4 \neq 2 = 2\det(I)$.

3. $Sv_1 = 0, Sv_2 = 0, Sv_3 = 2w_1, Sv_4 = 6w_2$. Then the matrix is:

$$\begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

4. Do Problem 2 from 7.2.

   If $v''(x) = 0$ then integrating gives $v'(x) = c$ and integrating again gives $v(x) = cx+d$. In terms of the previous problem, this is the span of the vectors v_1 and v_2, which is exactly the nullspace of B.

5. Do Problem 3 from 7.2.

   Adding a zero row to A gives

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

   For this new A, we have $A^2 = B$. We need to use the same output and input basis; that way, A^2 really is applying A twice.

6. Do Problem 4 from 7.2.

   Taking A, B as above, we get that AB is

$$\begin{pmatrix} 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

   The fourth derivative matrix is zero because for each of $1, x, x^2, x^3$ the fourth derivative is the zero function.

7. Do Problem 20 from 7.2.

   (a) We may take $w_2 = 1 - x^2$. This may be found by inspection or by setting up a system of equations for the coefficients A, B, C.

   (b) Note $w_3(x) = w_1(-x) = (x^2 - x)/2$.

   (c) $y = 4w_1 + 5w_2 + 6w_3$.

8. Do Problem 21 from 7.2.

   To go from w_1, w_2, w_3 to v_1, v_2, v_3 we simply use the matrix whose columns are w_1, w_2, w_3. Then the inverse matrix goes back.

9. Do Problem 11 from 8.1. (It seems that in some version of the 4th edition of the book, this problem is Problem 10, we want you to do the problem starting with the words "Find the displacements..."). You are only asked to do the fixed-fixed case.

   MATLAB PROBLEM, SEE MATLAB SOLUTIONS

10. Free points !

    Yay!